

A Comparative Study of Fuzzy Optimization through Fuzzy Number

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Abstract

This paper presents a model for analyzing fuzzy reliability of the system using fuzzy number mathematical operation, where a triangular fuzzy number represents the stability of each computer system. Because the method is using simple fuzzy mathematical operation of fuzzy numbers instead of difficult arithmetic and logical of intervals. We are introducing a rule-based approach for estimating precisely all the broadest collections of blurry contextual disparities, considered an expansion to fuzzy relational coefficients, fuzzy relational inequalities can be extended to other fuzzy logic areas.

Keywords- Fuzzy optimization; triangular fuzzy number; fuzzy logic.

1. Introduction

Operational research is already properly approved as applicable to the strengthening optimization. LPP help to make efficient use successful mechanisms (Chandrawat et al., 2017). This offers more resources to meet the current environment (Singh and Dhiman, 2017). It's dealing with the big issue of highest profit. Optimization is an analysis technique used for optimization. Optimization is a mathematical method in use for objective function (Singh and Dhiman, 2018). It is a scientific theory to sales management resolving that surfaced during the Second World War, when maximize the operation of a number of variables based on a set of difficulties (Kandel and Byatt, 1978). Optimization is a theoretical method for solving practical and functional problem of a linear equation, in which the function is limited inequalities. The real image of the LPP can be described by;

$$\text{Maximum } \Omega = \sum_{j=1, i=1}^m A^* y_j \quad (1)$$

Subject to

$$\sum_{j=1, i=1}^{n, m} C_{ji} * y_i \leq d * \forall y_i \geq 0$$

Fuzzy relationships, which are often found in various fields, such as mathematics, decision-making, and cluster analysis (Zadeh, 1965), are specific circumstances of L-relations where L is the standard sequence [0, 1] (Rasheed and Sarhan, 2019).

Fuzzy logic is already adjusted for different areas; from linear programming to AI., it has already been required to assist the computer to decide the variations between data that are not correct nor incorrect (Negoiță and Ralescu, 1975). If there is anything measurements of the human reasoning process such as darkness, some brightness etc. Fuzzy sets extend modernist sets, as the measurement processes (aka unique systems) of classical sets are specific cases of the objective function of fuzzy sets, if maybe it only takes values 0 or 1. Classical ions sets are commonly referred to as triangular fuzzy numbers in fuzzy logic (Mendel, 2003). The fuzzy number principle are used in a broad range of domains, like bioinformatics, that the knowledge is incorrect or imperfect (Liu et al., 2021).

Fuzzy logic is a type of multi-valued theory in which variable realized can be any real number between 0 and 1 including of it. It is used to resolve the theory of false statement, in which the result of truth can vary from truly true to completely false. In comparison, in Boolean logic, only integer values 0 or 1 can be the truth values of variables (Kumar et al., 2021).

2. Literature Review

Within the general Linear programming problem, the variables stay unchanged but we may assume that such quantity of limitation stays unchanged, however if the price of the rise or drop below a certain period in time, so this specific linear programming problems is known the uncertain Linear programming problems its member rank for the steadily increased value is implemented by fuzzy linear programming problems methods. This participation role consists can achieve best outcomes within the bottom and top limit scope of the linear programming problems. So, our initial issue would be turned in to equal, crisp query. After that, a crisp deal only with word that is defined so there is no fuzziness. Fuzzy suggests a loss of clearness and any idea that is unclear in every way. For example, the level of satisfaction for the fantasy character might be different in different study social characters. Usually, in the mathematical logic, we research the characteristics of the sets or the items that do not relate to the set but to the fuzzy set. For e.g., whenever the characteristic relates to the group, people classify membership degree 1 to it, if not 0 because in fuzzy logic, researchers can attach standard approach between 0 and 1. Zadeh (Zadeh, 1965) first decided to introduce the fuzzy logic idea. If this fuzzy logic is usable to linear decision making, then FLP came into being. Through using FLP, we can calculate improvements in other decision variables. There are many everyday situations requests from fuzzy linear programming, like business and technology online methods. The nature of optimization techniques and FLP some researchers have developed methods for solving this problem in some modern system using intuitive fuzzy number, and LPP operations have been implemented. Throughout this paper, we use the hexagonal fuzzy number linear programming problems to come to terms with either the predictive increase (pi) throughout the essential quality (bi) of classical optimization and discuss the results with the specified membership degree. Triangle

fuzzy sets can be used to interpret feasible uncertainties and missing data in decision-making, probability of the risk and optimization techniques (Türk, 2021).

3. Preliminaries

Definition 1

A α -cut of a fuzzy group of X is the set where the membership values in X , are higher than or equal to α , is indicated by \tilde{X}^α

$$\tilde{X}^\alpha = \{U \mid \lambda_x(U) \geq \alpha, u \in U\} \tag{2}$$

Definition 2

Let U be the universal space and a Fuzzy set \tilde{X} , a set in which each element of the set U is linked to a membership grade specified as:

$$\tilde{X} = \{(U, \lambda_x(U)) : u \in U, \lambda_x(U) \rightarrow [0,1]\} \tag{3}$$

Definition 3

A powerful α -cut of a fluffy set of U is the set in which the membership values in X , are higher than α , is indicated by $\tilde{X}^{\alpha+}$

$$\tilde{X}^{\alpha+} = \{U \mid \lambda_x(U) > \alpha, u \in U\} \tag{4}$$

Definition 4

The height of a fuzzy set denoted by $h(X)$ is defined as the elements contained in that set's largest membership values.

Definition 5

If a fuzzy set $h(X) = 1$ then fuzzy set A is called normal.

Definition 6

If the given condition is satisfied then the set is called convex fuzzy set.

$$\lambda_x \{\mu u_1 + (1 - \mu)u_2\} \geq \min \{\lambda_x(u_1), \lambda_x(u_2)\}, \text{ where } 0 \leq \mu \leq 1, u_1, u_2 \in U \tag{5}$$

Definition 7

The fuzzy number $P = \langle p, q, r \rangle$ are called triangular fuzzy number, if the membership function λ_P are defined

$$\lambda_{B(x)} = \begin{cases} \frac{y-b}{a-b}; b \leq y < a \\ 1; y = a \\ \frac{d-y}{d-a}; a \leq y < d \\ 0; \text{otherwise} \end{cases} \quad (6)$$

$$B_{\alpha} = [b, d] = [(a-b)_{\alpha} + b, -(d-a)_{\alpha} + d] \quad (7)$$

4. Mathematics Operation

It is a fundamental method of mathematics operation; that is addition, subtraction, multiplication and division.

Let P and Q are two fuzzy numbers.

$$P = [p_1, p_3] \quad Q = [q_1, q_3] \in R \quad (8)$$

(a) Addition of Two Fuzzy Set

$$P + Q = [p_1, q_3] + [q_1, q_3] \quad (9)$$

$$[p_1 + q_1, q_3 + p_3]$$

(b) Subtraction of Two Fuzzy Set

$$P - Q = [p_1, p_3] - [q_1, q_3] \quad (10)$$

$$[p_1 - q_1, p_3 - q_3]$$

(c) Multiplication of Two Fuzzy Set

$$P(.)Q = [p_1q_1 \& p_1q_3 \& p_3q_1 \& p_3q_3] \quad (11)$$

$$[p_1q_1 \text{ or } p_1q_3 \text{ or } p_3q_1 \text{ or } p_3q_3]$$

(d) Division of Two Fuzzy Set

$$P/Q = [p_1/q_1 \& p_1/q_3 \& p_3/q_1 \& p_3/q_3] \quad (12)$$

$$[p_1/q_1 \text{ or } p_1/q_3 \text{ or } p_3/q_1 \text{ or } p_3/q_3]$$

5. Mathematical Modeling

The graphical LPP methodology uses the maximum or minimum critical points of the objective function line (Torra, 2010) and the possible regions to solve problem the graphical methodology is used for solving LPP involving two variable decision y, x and instead of y_1, y_2



5.1 Classical Linear Programming

The problem with classical Linear Programming is the finding under limitations described by Linear equations or inequities.

$$\text{Maximum/Minimum } d_1y_1 + d_2y_2 + \dots + d_my_m \tag{13}$$

$$b_{11}y_1 + b_{12}y_2 + \dots + b_{1m}y_m \leq a_1$$

$$b_{21}y_1 + b_{22}y_2 + \dots + b_{2m}y_m \leq a_2$$

.....

$$b_{n1}y_1 + b_{n2}y_2 + \dots + b_{nm}y_m \leq a_n$$

$$y_1, y_2, \dots \geq 0$$

The maximized or minimized variable is known as objective function. Let us assume it is β . The expression of the question can be completed to use this symbol.

$$\text{Minimum } \beta = d_y \tag{14}$$

$$\beta_y \leq a$$

$$y \geq 0$$

5.2 Fuzzy linear programming

The following forms are the most particular form of linear fuzzy programming.

$$\text{Maximum } \sum_{i=1}^m D_i Y_i \tag{15}$$

Subject to

$$\sum_{i=1}^m B_{ji} Y_i \leq A_j (j \in M_m)$$

$$y_i \geq 0 (i \in M_m)$$

Case 1 Fuzzy LPP where on the right side only.

$$\sum_{i=1}^m D_i Y_i \tag{16}$$

Subject to

$$\sum_{i=1}^m b_{ji} Y_i \leq A_j (j \in M_m)$$

$$y_i \geq 0 (i \in M_n)$$



Case 2 Fuzzy LPP where on the right side just the function is the limits matrix the fuzzy numbers.

$$\sum_{i=1}^m d_i y_i \tag{17}$$

Subject to

$$\sum_{i=1}^m B_{ji} Y_i \leq A_i (j \in M_n)$$

$$y_i \geq 0 (i \in M_m)$$

Usually, fuzzy LPP will first be translated into analogous simple or anti linear problems which will then be solved by usual methods the final results of fuzzy LPP is therefore real values representing a solution in the form of the fuzzy numbers used. Here is the usual $A_i(y)$ from was used for $A_j(j \in M_m)$ fuzzy number.

$$A_i(y) = \begin{cases} 1 & \text{when } y \leq a_j \\ \frac{a_j+q_j-y}{q_j} & \text{when } a_j < y < a_j + q_j \\ 0 & \text{when } a_j + q_j \leq y \end{cases} \tag{18}$$

In which $y \in S$ we next measure the degree of $\beta_j(y)$ by which y approaches the i^{th} limit with ($i \in M_n$) the method for any vector $y = y_1 y_2 \dots, y_n$.

$$C_j(Y) = A_j \sum_{i=1}^m b_{ji} Y_i \tag{19}$$

This is a fuzzy set on S^m and their junction $\cap_{j=1}^n D_j$ is a fuzzy set to function the fuzzy set of optimal value is now calculated. Last it is achieved by first measuring the high and low limits of the optimal values β_k . By solving the classical linear programming problems and can be achieved by trying to reduce the optimal values.

$$\text{Maximum } \beta = d_y \tag{20}$$

Subject to

$$\sum_{i=1}^m B_{ji} Y_i \leq A_i (j \in M_n)$$

$$y_i \geq 0 (i \in M_m)$$

A similar linear programming problem occurs in the upper limit for the best possible value β_k in which each is substituted by $(a_j + q_j)$.

$$\text{Max } \beta = d_y \tag{21}$$

Subject to

$$\sum_{i=1}^m B_{ji} Y_i \leq A_i (j \in M_n)$$

$$y_i \geq 0 (i \in M_m)$$

The fuzzy set of feasible variables H is then defined by the fuzzy S^m subset.

$$H(Y) = \left\{ \begin{array}{l} 1 \text{ when } y \leq a_j \\ \frac{d_j + \beta_k}{\beta_v - \beta_k} \text{ when } \beta_k \leq D_y \leq \beta_v \\ 0 \text{ when } D_Y \leq \beta_k \end{array} \right\} \quad (22)$$

The problems have the standard problems of optimizing.

$$Y (\beta_k - z_k) - d_y \leq -\beta_k \quad (23)$$

Subject to

$$\gamma_{qk} + \sum_{i=1}^m B_{ji} Y_i \leq a_j + q_j (j \in M_n)$$

$$Y, y_i \geq 0 (i \in M_m)$$

5.3 Fuzzy Optimal Solution to Fully Fuzzy Linear Programming Problems

$$\text{Max / Min } \tilde{Z} = \sum_{i=1}^m \tilde{\Omega}^T \otimes \tilde{Y}_i \quad (24)$$

Subject to

$$\tilde{B} \otimes \tilde{Y} = \tilde{a},$$

\tilde{Y} is a non – negative fuzzy number.

The proposed method takes the following steps:

STEP-1

Putting

$$\tilde{\Omega}^T = [\tilde{\Omega}_i]_{1 \times m}, \tilde{Y} = [\tilde{y}_i]_{m \times 1}, \tilde{B} = [\tilde{b}_{ji}]_{n \times m}, \tilde{a} = [\tilde{a}_j]_{n \times 1} \quad (25)$$

we get

$$\text{Max \ Min } \left(\sum_{i=1}^m \tilde{\Omega}_i \otimes \tilde{y}_i \right)$$

$$\text{subject to } \sum_{i=1}^m \tilde{b}_{ji} \otimes \tilde{y}_i = \tilde{a}_j \forall i \in (N)$$

\tilde{y}_i is the fuzzy number.

STEP 2

If all the limits $\tilde{c}_n, \tilde{b}_n, \tilde{a}_n, \&, \tilde{x}_{mn}$ is showed by triangular fuzzy number $(f_n, g_n, h_n), (a_n, b_n, c_n), (x_{mn}, y_{mn}, z_{mn})$, and (p_n, q_n, r_n) showed, then the FFLP problem.

$$\text{Max(or Min) } R \left(\sum_{n=1}^j (f_n, g_n, h_n) \otimes (a_n, b_n, c_n) \right) \tag{26}$$

Subject to $\sum_{n=1}^j (a_n, b_n, c_n) \otimes (x_{mn}, y_{mn}, z_{mn}) = (p_j, q_j, r_j) \forall i = \text{natural number}(N)$
 (x_n, y_n, z_n) is a positive triangular fuzzy number.

STEP 3

Consider $(x_{mn}, y_{mn}, z_{mn}) \otimes (a_n, b_n, c_n) = (m_{mn}, n_{mn}, o_{mn})$

$$\text{Maximize (or Minimize) } \Leftrightarrow R \left(\sum_{n=1}^j (f_n, g_n, h_n) \otimes (a_n, b_n, c_n) \right) \tag{27}$$

Subject to

$$\sum_{n=1}^j (m_{mn}, n_{mn}, o_{mn}) = (p_n, q_n, r_n) \forall i = N$$

(a_n, b_n, c_n) is a positive triangular fuzzy number.

STEP 4

$$\text{Max(or min) } R \left(\sum_{j=1}^n (f_j, g_j, h_j) \otimes (a_j, b_j, c_j) \right) \tag{28}$$

Subject to

$$\sum_{n=1}^j m_{mn} = f_i, \forall i = N$$

$$\sum_{n=1}^j m_{mn} = f_i, \forall i = N$$

$$\sum_{n=1}^j m_{mn} = f_i$$

5.4 S, L, R METHOD

Suppose all the fuzzy number is triangular. We can write $B = \langle s, l, r \rangle$

$$\text{Maximum} \sum_{i=1}^m d_i y_i \tag{29}$$

Subject to

$$\sum_{i=1}^m \langle s_{ji}, l_{ji}, r_{ji} \rangle y_{ji} \leq \langle u_j, l_j, r_j \rangle \langle j \in M_n \rangle$$

$$y_i \geq 0 \langle i \in M_n \rangle$$

Where $B_{ji} = \langle s_{ji}, l_{ji}, r_{ji} \rangle$ & $A_{ji} = \langle u_j, v_j, w_j \rangle$ is fuzzy number. $B \leq A$ iff and only if

Maximum (B, A) = A. it proved for any 2 triangular fuzzy numbers.

$B = \langle s_2, l_2, r_2 \rangle$ and $A = \langle s_1, l_1, r_1 \rangle$, $B \leq A$

if and only if $s_2 \leq s_1, s_2 - l_2 \leq s_1 - l_1$ and $s_2 + r_2 \leq s_1 + r_1$

$$\langle s_2, l_2, r_2 \rangle + \langle s_1, l_1, r_1 \rangle = \langle s_2 + s_1, l_2 + l_1, r_2 + r_1 \rangle \& \langle s_2, l_2, r_2 \rangle y = \langle s_{2y}, l_{2y}, r_{2y} \rangle$$

Where y is non- negative number.

$$\text{Maximum} \sum_{i=1}^m d_i y_i \tag{30}$$

Subject to

$$\sum_{i=1}^m s_{ji} y_i \leq p_j$$

$$\sum_{i=1}^m (s_{ji} - l_{ji}) y_i \leq p_j - q_j$$

$$\sum_{i=1}^m (s_{ji} + l_{ji}) y_i \leq p_j + q_j \langle j \in M_n \rangle$$

$$y_i \geq 0 \langle i \in M_n \rangle$$

6. Numerical experiment

Example 1 Let suppose that a registered firm is making only two products and assume it has name A_1 and A_2 having cost \$.30 and \$.40 per unit gain as shown in Figure 1. The each unit of product A_1 needs twice as many labor hours as each product A_2 as shown in Figure 2. The total availability of labour are 500 per day that too minimum and suppose it need to be extended up to 600 hours per day because of arranging some important overtime work as shown in Figure 3. The supplying of material is ok for 400 units for the both A_1 and A_2 product in a day but it can be possible to extend it 500 units in a day as per previous experiences as shown in Figure 4. The question is now how much units of product A_1 and A_2 could be made in a day to have maximum gain?

Solution: Let y_1, y_2 in a notation of the numbers of units of the products A_1 and A_2 produced in a day a solution can be by using FLPP given below.

$$\text{Maximum } \Omega = 3y_1 + 4y_2 \quad (31)$$

Subject to

$$\begin{aligned} y_1 + y_2 &\leq C_1 \\ 2y_1 + y_2 &\leq C_2 \\ y_1, y_2 &\geq 0 \end{aligned}$$

Where C_1 defined as

$$C_1(y) = \begin{cases} 1 & \text{when } y \leq 400 \\ \frac{500 - y}{100} & \text{when } 400 < y \leq 500 \\ 0 & \text{when } 500 < y \end{cases} \quad (32)$$

Where C_2 is defined as

$$C_2(y) = \begin{cases} 1 & \text{when } y \leq 500 \\ \frac{600 - y}{100} & \text{when } 500 < y \leq 600 \\ 0 & \text{when } 600 < y \end{cases} \quad (33)$$

Further solving we need upper limits and lower.

$$\Omega_l = 130 \text{ and } \Omega_u = 160 \quad (34)$$

$$A_1 \text{ Maximum } \Omega = .3y_1 + .4y_2$$

Subject to

$$\begin{aligned} y_1 + y_2 &\leq 400 \\ 2y_1 + y_2 &\leq 500 \\ y_1, y_2 &\geq 0 \end{aligned}$$

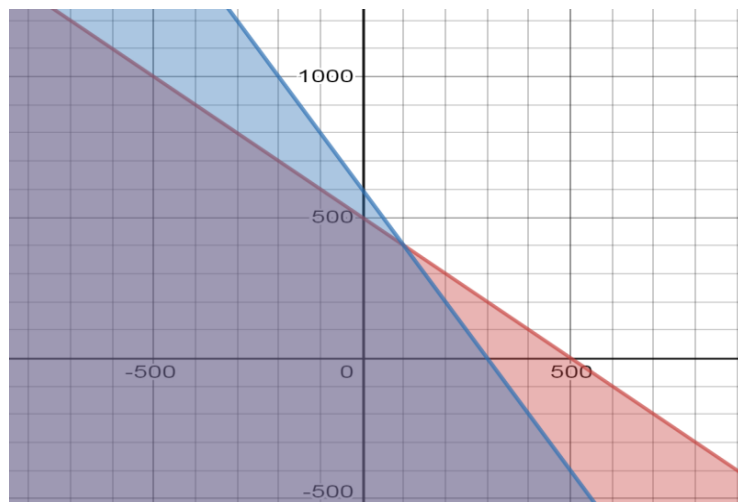


Figure 1: Solution of equation number (34).

$$A_2 \text{Maximum } \Omega = .3y_1 + .4y_2 \quad (35)$$

Subject to

$$\begin{aligned} y_1 + y_2 &\leq 500 \\ 2y_1 + y_2 &\leq 600 \\ y_1, y_2 &\geq 0 \end{aligned}$$

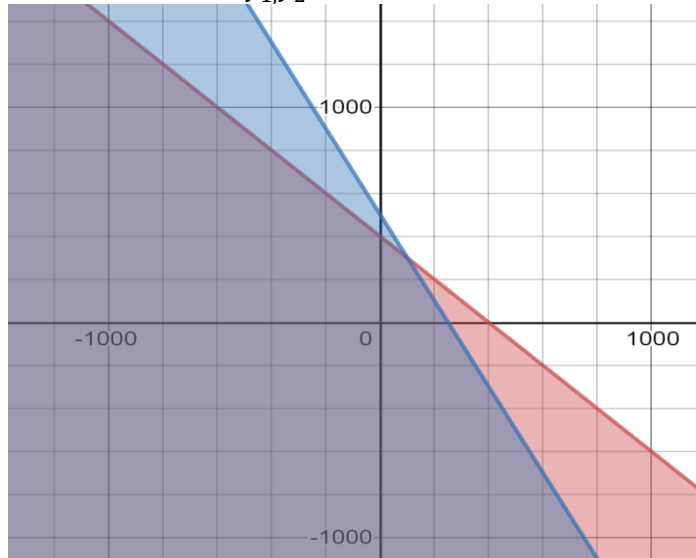


Figure 2: Solution of equation number (35).

Example 2 Suppose the following fully fuzzy programming problems and solved as given below.

$$\max(2,7,8) \otimes \tilde{P}_1 \oplus (1,4,9) \otimes \tilde{P}_2 \quad (36)$$

Subject to

$$(1,4,5) \otimes \tilde{P}_1 \oplus (2,3,4) \otimes \tilde{P}_2 = (7,17,33)$$

$$(-2,2,3) \otimes \tilde{P}_1 \oplus (2,4,5) \otimes \tilde{P}_2 = (2,18,34)$$

\tilde{P}_1, \tilde{P}_2 are non-negative fuzzy number.

Solution: Consider $\tilde{P}_1 = (P_1, Q_1, R_1)$ and $\tilde{P}_2 = (P_2, Q_2, R_2)$ then fully fuzzy linear programming problem be written below as.

$$\text{Max}(2,7,8) \otimes (P_1, Q_1, R_1) \oplus (1,4,9) \otimes (P_2, Q_2, R_2)$$

Subject to

$$(1,4,5) \otimes (P_1, Q_1, R_1) \oplus (2,3,4) \otimes (P_2, Q_2, R_2) = (7,17,33)$$

$$(-2,2,3) \otimes (P_1, Q_1, R_1) \oplus (2,4,5) \otimes (P_2, Q_2, R_2) = (2,18,34)$$

(P_1, Q_1, R_1) and (P_2, Q_2, R_2) are non-negative fuzzy number.

Using step 3, the above FLLP problems.

Subject to
$$\text{Max } R (2P_1 + P_2, 7Q_1 + 4Q_2, 8R_1, 9R_2)$$

$$(P_1 + 2P_2, 4Q_1 + 3Q_2, 5R_1, 4R_2) = (7, 17, 33) \tag{37}$$

$$(-2P_1 + 2P_2, 2Q_1 + 4Q_2, 3R_1, 5R_2) = (2, 18, 34) \tag{38}$$

Using step 4

$$\text{Max } (2(2P_1 + P_2) + \frac{1}{4}(7Q_1 + 4Q_2) + (8R_1, 9R_2))$$

$$P_1 + 2P_2 = 7 \tag{39}$$

$$7Q_1 + 4Q_2 = 17 \tag{40}$$

$$8R_1 + 9R_2 = 33 \tag{41}$$

$$-2P_1 + 2P_2 = 2 \tag{42}$$

$$2Q_1 + 4Q_2 = 18 \tag{43}$$

$$3Q_2, 5R_1 = 34 \tag{44}$$

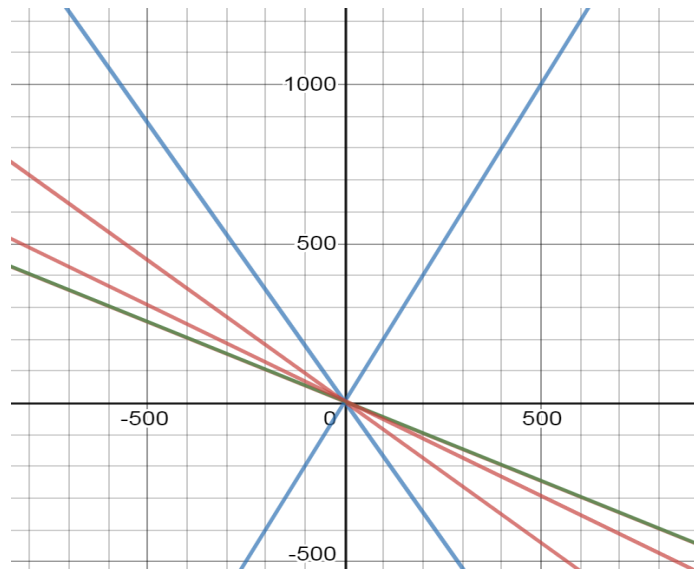


Figure 3: Solution of step number 4.

Example 3(Problem on Fully fuzzy linear programming problem)

$$\text{Maximum } \Omega = 6y_1 + 3y_2 \tag{45}$$

Subject to

$$\langle 6,4,2 \rangle y_1 + \langle 7,5,3 \rangle y_2 \leq \langle 12,6,3 \rangle$$

$$\langle 6,4,2 \rangle y_1 + \langle 3,2.5,3 \rangle y_2 \leq \langle 24,5,8 \rangle$$

$$y_1, y_2 \geq 0$$

Solution: Using SLR method

$$\text{Maximum } \Omega = 6y_1 + 3y_2 \tag{46}$$

Subject to (47)

$$6y_1 + 7y_2 \leq 12 \tag{48}$$

$$6y_1 + 3y_2 \leq 24 \tag{49}$$

$$2y_1 + 2y_2 \leq 6 \tag{50}$$

$$4y_1 + 5y_2 \leq 19 \tag{51}$$

$$8y_1 + 10y_2 \leq 15 \tag{52}$$

$$y_1, y_2 \geq 0 \tag{53}$$

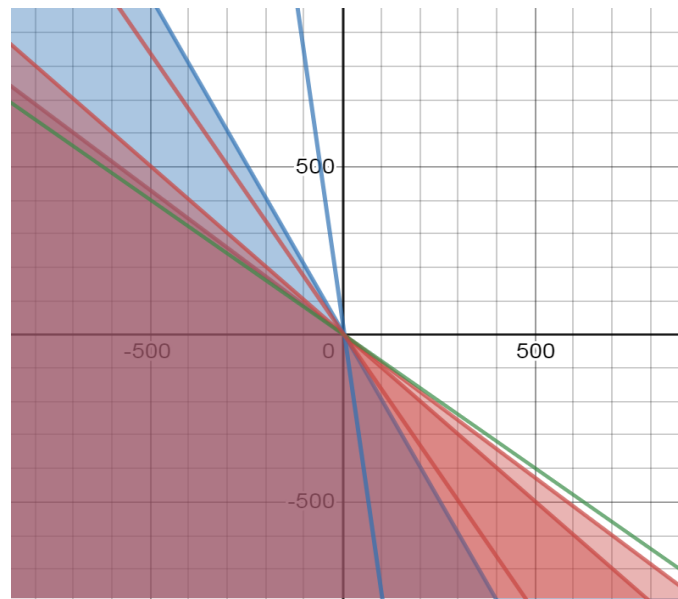


Figure 4: Graphical solution of equation (45).

7. Conclusion

The benefits of Linear Programming System Linear programming always apply in cases at which parameters and optimal solution are static where they'll be represented as straight line variables. This approach will not be used in daily - life situations where parameters or parameter values are not linear. Things like complexity and resources are not taken into account. The benefits of Linear Programming System Linear programming always apply in cases at which parameters and optimal solution are static where they will be represented as straight line variables. This approach will not be used in real life situations where parameters or parameter values are not linear. Things like complexity and resources are not taken into account. The user's decision-making technique is

becoming more objective but less subjective. Such devices will be in great supply in a factory, for instance, while others may make false claims idle for certain period.

Conflict of Interest

The authors confirm that there is no conflict of interest to declare for this publication.

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